

HELPING STUDENTS TO CONNECT SUBTRACTION STRATEGIES IMPROVES MATHEMATICAL REASONING FOR STUDENTS AND TEACHERS

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ABSTRACT

After administering an end of unit assessment written by the school's math program, teachers of three second grade classes in a New York City school noticed a majority of the students had not demonstrated mastery of subtracting two, two-digit numbers. The teachers worked with the school's math coach to implement an instructional unit that required students to make connections among three different subtraction strategies. Implementation of the unit resulted in improved mathematical reasoning for students and teachers in addition to an improvement in students' subtraction skills.

INTRODUCTION

In a New York City public school, second grade students were struggling with subtraction. This particular second grade had three very different classes, each was set up for a different purpose, and was taught using dissimilar teaching styles. Midway through the school year an assessment, created by the school's math program, containing three subtraction problems was given to the students. The percent of students that answered these three specific questions correctly was calculated as 55% for Class A, 42% for Class B, and 25% for Class C. The teachers of Class C asked the school's math coach¹ (the author of this article) to recommend effective instructional strategies for subtraction. The math coach suggested including the teachers of Class A and B in the discussion, and the collaborative action research group was formed. The group chose to focus their study on strategies designed to increase the percent of students mastering abstract subtraction of

¹ Math coaches are math teachers hired to provide professional development in mathematics.

two, two-digit numbers with and without trading². The results of which demonstrated an increase in the percent of student mastering subtraction, as well as an improvement in the mathematical reasoning abilities on the part of both students and teachers.

Implementing this approach was going to take a significant amount of time. The need for simultaneous implementation of the process and evaluation of the process was needed to determine if the process should be repeated in the future. The process of deep inquiry into one's practices in service of moving towards an envisioned future is known as action research (Reil, 2010). "Collaborative action research may include as few as two teachers or a group of several teachers and together interested in addressing a classroom or department issue" (Ferrance, 2000, p.1). This article chronicles the collaborative action research group's experience as they moved through the five stages of an action research cycle described by McNiff as: identify an area of practice to be investigated, imagine a solution, implement a solution, and evaluate the solution, change practice in light of the evaluation (2002).

STAGE 1: IDENTIFY AN AREA OF PRACTICE TO BE INVESTIGATED

The 53 students attended the same elementary school, but were separated into three different classes. The three classes followed the same pacing guide and used the same mathematics program purchased by the school. However, the teachers of each class differed stylistically and philosophically in their approach to teaching mathematics. Class A was a traditional regular education class with one teacher. Prior to the implementation of the action plan, this class had no experience with the concrete or representation strategies for subtraction. This class did have some experience using the traditional subtraction algorithm before the baseline data was collected. Class B was a Collaborative-Team Teaching class. Nearly 50% of the students had an Individual Education Plan (IEP) for mild learning disabilities, such as delayed speech and language skills. The remaining students were of average to above average intelligence.

Prior to collection of the baseline data, most of the students were comfortable using base 10 blocks to represent numbers and had a conceptual understanding of place value and some exposure to the concept of subtraction. None of the students in Class B was exposed to the traditional subtraction algorithm prior to collection of the baseline data. Class C was a Collaborative-Team Teaching class that had 25% of the students identified as having Asperger's Syndrome. The remaining 75% of the class was comprised of students with mixed ability levels.

Prior to the study, the students in this class had no experience with the formal subtraction algorithm. They had a strong understanding of place value and some exposure to the concept of subtraction. At approximately the same time of the school year all three classes administered the same end of unit assessment provided by the math program. Three

² Trading is defined as the process of exchanging one group of ten for a group of ten ones. For example, 36 can be represented as 3 tens and 6 ones ($30 + 6$) but if 36 is regrouped, then it is represented as 2 tens and 16 ones ($20 + 16$). To deal with this problem an action plan was created and implemented

questions of the test required students to subtract two, two-digit numbers. Teachers defined mastery of subtraction as the ability to find the correct difference for all three subtraction examples. According to this definition, the percent of students demonstrating mastery in Classes A, B and C was 55%, 42% and 25% respectively. These results were used as the baseline data for the action research.

In her article “Nine Ways to Catch Kids Up,” Marilyn Burn states that it is important to help students make connections among mathematical ideas so they do not see them as isolated facts (2007). Furthermore, Rosenshine notes it is easier to assimilate new information as well as use prior knowledge for problem solving when one has more connections and stronger ties between connections in a given knowledge structure (1995). If this is the case, what type of connections do we expect second grade students to make when they are trying to master an understanding of subtraction? Research suggests that students who can use the Concrete-Representation-Abstract Sequence have demonstrated success when learning concepts such as subtraction and algebra (Witzel, 2003; Flores, 2009). In this method students are explicitly taught to solve a mathematical problem concretely with the use of manipulatives, followed by the use of student-created drawings for a representation strategy. Lastly, students are taught to solve the same type of problems abstractly (using numbers and math symbols). The success of this method suggests that there are important connections to be made among the three different representations. When students use the relationships among mathematical content they extend their ability to apply concepts and skills more effectively (National Council of Teachers of Mathematics, 2000). Therefore, the purpose of this action research was to find ways to help students make mathematical connections among the concrete, pictorial, and the abstract subtraction strategies, and to use these connections to master subtraction of two, two-digit numbers.

STAGE 2: IMAGINE A SOLUTION

Developing an action plan started with a meeting at which time each teacher explained: their current teaching strategies, their reasons for choosing their approaches, and the results they had seen in their students up to that point. The majority of students used number lines, one-hundred charts, or counting strategies to solve subtraction problems. Some students in one class had demonstrated the ability to use the traditional subtraction algorithm. Even with the differences in teaching styles and student population, many commonalities among the students were noticed. A majority of students had a difficult time deciding when to use a particular strategy. Students’ inability to connect new problems to previously solved problems resulted in an inability to recall prior knowledge that would be useful in selecting a subtraction strategy. Often students would automatically use the last strategy learned, or they would wait to be prompted for a particular strategy by the teacher. Because students demonstrated mastery of adding numbers abstractly, the teachers and math coach concluded students could master a problem solving strategy, but had difficulty recognizing when it was appropriate to use the strategy. The following three conditions were identified as necessary for students to connect a problem with the appropriate problem solving strategy effectively.

1. Students need to connect various representations to each other explicitly.

The *Principles and Standards for Teaching Mathematics* states: “[in order to become deeply knowledgeable...students will need a variety of representations that support their understanding” (National Council of Teachers of Mathematics, 2000, p. 68)]. The three subtraction strategies used were: concrete (through the use of base 10 blocks), pictorial (student-created or pre-drawn), or abstract (through the use of numeric or symbolic notations). Each strategy uses a different representation to depict a different nuance of the same concept. As Van de Walle and Lovin explain, “[t]he more existing ideas that are used to give meaning to the new one, the more connections will be made. The more connections [that are] made, the better the new idea is understood” (2006, p.2).

2. Students need to spend a significant amount of time working at solving problems using the concrete and representation strategy before learning the abstract strategy.

A common teaching practice is to use manipulatives to concretely model subtraction problems as a way to demonstrate the conceptual underpinnings of a subtraction strategy. This is followed by the assertion the abstract strategy is “the way” to solve the problem. Teachers tend to think that using manipulatives will help students “see” the mathematical ideas, as though “seeing” would automatically produce understanding (Schram, 1989, p.10). Even if it were possible to develop understanding through passive observation, it would be impossible for a teacher to know either the depth of the student’s conceptual understanding or if the student has made any connections that will be useful in future problem solving. For a teacher to have an accurate knowledge of such things, students need to demonstrate their own understanding and skills. Students need to describe the properties of numbers in their own words and symbolic notation (Ketterlin-Geller, Jungjohann, Chard and Baker, 2007). To do this the students need time to master the concrete and representation strategies before learning the abstract method.

3. Students need time to connect problems and form generalizations that will help in future problem solving.

The developmental process is characterized by generalization, and the importance of generalization cannot be overstated (Fosnot and Dolk, 2001). To be able to teach students that there are many problems but only a few strategies is a very powerful concept. In order for students to connect one strategy to many problems it is important for students to: be given time to study various problems, find the similarities among the group of problems, and analyze why a given strategy is applicable to all the problems in the group. As Burns explains, “when you understand why, your understanding and skills can be applied more easily to new tasks” (Burns, 2000, p. 151).

STAGE 3: IMPLEMENT A SOLUTION

PROVIDE PROFESSIONAL DEVELOPMENT

What teachers expect students to know is related to [the teachers'] own knowledge (Ma 1999). We felt with this project, if the teachers' expectations for their students were to change, the teachers' understanding of subtraction would evolve. The math coach began the professional development sessions by asking the teachers to look at the different strategies for subtraction such as: the concrete use of base 10 blocks, representation through student created drawings and the abstract use of mathematical numbers/symbols. The teachers and the math coach discussed the different nuances that each strategy demonstrated. Next, teachers were asked at what point in each strategy is the problem solver confronted with the question, 'Do I need to trade?' Subsequently, a discussion regarding appropriate justification for trading ensued. This included a dialogue about the specific language the teachers would expect the students to use when justifying the need to regroup and how that language might differ when using concrete, representation, or abstract subtraction strategies.

Ultimately, the teachers and the math coach decided that students' ability to verbally justify the need for trading was a benchmark of achievement when working with the concrete or representation strategy. Furthermore, learning the abstract strategy for trading would not occur until students achieved this benchmark and verbally justify the need for it. Lastly, the teachers and the math coach separated subtraction into three groups of problems: subtraction of two, two-digit numbers without trading, multiples of ten minus a two-digit number, and subtraction of two, two-digit numbers with trading. Similarities of different problems of the same group were identified. Using the identified similarities, teachers formed generalizations that would be used in future problem solving opportunities. The professional development stage concluded with a discussion of appropriate vocabulary teachers would expect students to use during the study of each type of problem.

COLLABORATIVELY IMPLEMENT CONNECTION-BUILDING ACTIVITIES

Three separate classroom activities were designed to promote the formation of connections among various subtraction problems and the strategy used to solve them. The three activities were introduced to the students with the classroom teacher(s) and the math coach collaboratively. The first activity was to group students into partnerships; one student was called the "solver" and the second student the "recorder". During solver-recorder partnership work, each solver was given a subtraction problem and base 10 blocks to solve the problem concretely. The use of the base 10 blocks enabled students with limited mathematical communications skills to demonstrate why regrouping a ten for ten one was or was not necessary. As the solver explained/demonstrated the subtraction strategy, the recorder drew an original pictorial representation of this strategy. In the beginning, some partnerships had difficulty demonstrating the solutions to the same problem using both strategies. The teachers or the math coach would conference with the partnerships at this time and model the appropriate mathematical reasoning. Students were encouraged to use the same language. Having an opportunity to try out the mathematical language in partnerships allowed the students to build confidence in their communication skills before speaking in front of the whole class. One of the teachers in

Class B stated allowing students to practice the language in partnerships resulted in “more talk on this topic than any other topic this year.”

During this process it is important to use both manipulatives and pictorial representations that are appropriate to the age level and developmental level of the students (Witzel, 2003). In addition to the appropriateness of the pictorial representations, it is best when the pictures are drawn by the students. When students construct their own pictures it is a powerful learning experience because they must consider several aspects of mathematical ideas that are often assumed when pictures are pre-drawn for the students (Clements, 2004). The recorder could not solve the problem, but both solver and recorder had to agree the concrete and the representation solutions followed the same mathematical reasoning. The Concrete-Representation-Abstract method used in other studies teaches each method consecutively, with mastery of one stage required before instruction in the second strategy. For this action plan the decision to use the concrete and the representation strategies simultaneously was made to promote the formulation of connections between the two strategies in the students’ knowledge structure. During solver-recorder partnership work, the teacher and math coach would circulate among the students to monitor their progress. Upon mastery of those two subtraction strategies (concrete and representation) and use of the proper mathematical language to explain their process, students were taught to solve problems abstractly. Thus, the abstract strategy was used in addition to the concrete or representation strategy until the teacher felt that the student could explain the connections between the different strategies.

During the initial meeting with the math coach, teachers explained how they would ask their students to justify their thought process by asking questions, such as: “Why did you use that strategy?” or “Why do you think that strategy is correct?” Many students did not have an answer for these questions. Of the answers that were given, the most popular student responses were: “I picked that strategy because it would get me the right answer.” or “I know my strategy is correct, because the answer is right.” The teachers and the math coach agreed that connecting the correct answer to the justification of the strategy choice was not sound mathematical reasoning. The group decided that students would have to learn to consciously choose a strategy for a particular reason before attempting to solve problems.

The second activity was created to develop the practice of analyzing characteristics of the problem before solving it; it was called the Trade or No Trade game. In this game, students were given signs that said “Trade” and “No Trade”. The teacher would write a math problem on the white board. Students were given time to decide if trading would be necessary to solve the problem. If the students thought that they would regroup to solve the problem, they would hold up the “Trade” sign. If the students did not want to trade to solve the problem, they would hold up the “No Trade” sign. If called upon, students would have to justify their decision to the class. Students were not asked to solve the problem during this activity. Not requiring a solution kept the students focused on using the characteristics of the problem to make their decision.

The final activity was designed to help find connections among problems that used regrouping in the solution process, as well as those that did not. The teacher displayed a T-Chart and labeled the columns "Trade" or "No Trade". Students took turns selecting a piece of paper with a subtraction problem on it. The students had to explain to the class if regrouping was necessary to solve the problem; then, place the problem into the proper column. Once again, the students were not allowed to solve the problem to justify their answers. Once all problems were placed in a column on the T-chart, students had an opportunity to summarize their experience. Students looked at each list of problems and were asked, "What is the same about all the problems in the group?" Quickly the students began to generate responses, such as: "If the numbers in the ones columns match, the problem goes in the No Trade column." "If the top number [minuend] has more ones than the bottom number [subtrahend], it goes in the No Trade column." "When you have less ones than you have to remove, it [the problem] goes in the Trade column." This activity pushed students to find a mathematical reason that connected the separate problems as members of the same group. Next, students were asked how the similarity they found could be helpful when selecting a subtraction strategy. The discussion allowed students to form a generalization that explained why a particular strategy was applicable to all the problems in the group. The generalizations were used to justify mathematical reasoning in future problem solving opportunities.

STAGE 4: EVALUATE THE SOLUTION

When conducting collaborative action research in three classes establishing continuity is important. Several steps were taken to make sure there was a high degree of consistency in the instruction of the three classes. First, in the initial meeting, teachers agreed on the specific language teachers would use when introducing the activities, as well as the language students would be expected to use when justifying a strategy choice. Second, the math coach collaboratively taught the introduction of the three activities in all classes to increase consistency of instruction. Third, the math coach observed follow-up lessons and recorded observations to provide feedback to the teachers throughout the process. The observations, along with student work samples provided anecdotal data for the teachers to use to monitor the progress and process of their students throughout the study. Fourth, the group met during the process to share their experiences of completed lessons and to collaboratively plan the next steps. Fifth, the group used the same baseline and post-test assessment to measure student progress.

It is believed that the difference in posttest scores is due to the prior knowledge of and experience with base 10 blocks prior to the study. Students in Class A had a limited exposure to base 10 blocks prior to the study, while students in Class B and C had used base 10 blocks in a variety of activities. These students in Class B and C had the blocks in close proximity to their desks at every math class. Students were encouraged to take the blocks anytime they perceived the blocks to be useful. These students felt comfortable with the tool. They were familiar with the name and value of each block. The math coach noticed that students displayed a positive attitude towards using the blocks and transitioned smoothly and quickly from seeing the blocks as a tool for place value and addition to a tool for subtraction.

In Class B, the percent of students mastering subtraction had increased from 42% to 52%. However, the unique make-up of this class made it difficult to create an assessment that could accurately measure the improvement of the abilities of all students. Students without an IEP made a smooth transition from the concrete to the abstract. Once these students saw that the concrete and the representation strategy used a regrouping in the solution, they knew the abstract strategy would also use regrouping in its solution. Furthermore, many of these students who mastered subtracting two, two-digit numbers were able to transfer this knowledge to subtracting three-digit numbers.

In contrast, many students with an IEP did not automatically select regrouping as a strategy in the abstract solution even though they just used regrouping when they solved the problem concretely. Interestingly, Flores' study showed a similar phenomenon when students moved from the concrete phase to the pictorial phase (2009). It suggests that students who struggle in mathematics, not only compartmentalize each problem they solve, but also each solution. If this is the case, the need for teachers to make explicit connections during instruction becomes increasingly more important because there is a population of students who cannot independently make and apply connections that may seem obvious to their peers.

Students in Class C, made the smoothest and quickest transition from the concrete to the abstract representations, as well as the most significant improvement from the baseline to the final assessment. The percent of students mastering subtraction improved from 25% to 82%. Of the students who did not successfully solve all the subtraction problems on the posttest, only one student made errors in either the process of regrouping or the choice to select regrouping as a strategy. All other errors were the result of incorrectly recalling basic subtraction facts.

In contrast to her observations of Class B and C, the math coach observed that students in Class A demonstrated a strong resistance to using the base 10 blocks. Schram argued, "While teachers appreciate the role of manipulatives, they seem to view them as crutches rather than tools" (1989, p.11). For this reason, Class A rarely used manipulatives as a tool for discussion of mathematical ideas prior to the action plan. Lack of exposure to manipulatives transferred the teacher's belief that manipulatives were "crutches" to the students in her class. Students related the manipulatives to work they had done in first grade. Students who had mastered the algorithm saw using the manipulatives as a step backwards; they repeatedly questioned the need for "another way to subtract". Students who struggled with the algorithm saw using base 10 blocks as a signal to their peers that they were weaker math students than their classmates. Struggling students were hesitant to take advantage of the manipulatives, which were clearly in reach, unless specifically directed to do so. This demonstrated that the rush from manipulatives can be motivated by students as well as teachers.

Students in Class A exhibited a lack of flexibility throughout the course of the action plan. For example, when asked to solve $52-35$ using base 10 blocks some students, who previously demonstrated mastery of regrouping, subtracted 5 ones minus 2 ones which led

to a final answer of 23. Russell found, “decades of data show us that students in the US can learn to compute simple problems successfully without gaining understanding that they need to solve more complex problems” (2000, p. 156). Similarly, students in Class A that demonstrated mastery of subtraction on the baseline assessment could follow the steps of the given algorithm. However, the same students could not explain how regrouping affected the number or why regrouping was an appropriate for the problem solving process. In other words, students who learn the abstract procedure can do so without developing the flexibility to solve extensions of the problem (Carpenter, Franke, et al., 1997). For students to understand the abstract concepts more easily, Devlin explains, “we should start with what is familiar and concrete, and move gradually into the abstract” (2000, p.B5). Class A was the only class that had to attach conceptual understanding of subtraction to a pre-existing abstract understanding of subtraction. This proved to be a more difficult and time consuming way to develop a deep understanding of subtraction. After the same number of instructional hours, Class A had the most fragile grasp of the concept. This is the reason why in Class A, the percent of students demonstrating mastery on the posttest decreased from 55% to 44%.

Clearly defining the roles of each person in the solver-recorder partnership work pushed students to learn all three (concrete, representation and abstract) strategies. During the discussion between the solver and the recorder, students connected each aspect of the concrete solution with a specific part in the representation and the abstract strategies of the same problem. This was necessary, because when information is ‘meaningful’ to students, they have more points in their knowledge structures to which they can attach new information (Rosenshine, 1995). Requiring students to solve the same problem using two or three strategies helped students make connections between the abstract strategy and their existing knowledge structure developed through the concrete and representation problem solving phase. The teachers cited increased time on tasks and a decrease in the number of students waiting for teacher prompts as evidence that the students had developed a single more efficient knowledge structure for subtraction. The discussions observed by the teacher and the math coach provided the anecdotal evidence that proved students were connecting the strategies rather than merely learning two separate and distinct strategies.

Prior to this experience, many students could not justify the appropriateness of a strategy before the problem was solved. Once solved, students justified their strategy choice by pointing to a correct answer. Requiring students to justify the choice to trade before the problem was solved elevated the choice to a conscious decision rather than just another step in the procedure. The Trade or No Trade game helped students develop the habit of strategizing an approach to a problem before they even started solving it. Teachers noticed that students who could predict if trading would be a necessary part of the problem solving strategy were more likely to subtract correctly. Part one of the final assessment contained four problems where students had to state if trading would be needed in the solution. Students had to make this judgment without solving the actual problem. On part two, students had to solve three subtraction problems using the abstract strategy. On this assessment, 70% of the students who correctly identified whether or not regrouping was

necessary in part one, received a perfect score when they were asked to solve subsequent problems on part two.

The final activity of the action plan required students to sort subtraction problems into a T-chart labeled trade or no trade. This activity did not require students to solve the problems. While observing the students during the activity, teachers and the math coach noted the students' demonstrated varied but improved reasoning abilities. Even students with speech and language disabilities demonstrated the ability to articulate a reason why trading was or was not necessary for a given problem. Other students were able to analyze a fellow student's mathematical argument about the strategy choice, accept, reject, or clarify it. For example, one student discovered, "[e]very time there is a zero in the ones place in the first number [minuend], I will have to regroup." This was then clarified by another student who added, "not if there is a zero in the ones place in the bottom number [subtrahend]."

Ma found teachers who expect students to merely learn mathematical procedures tend to have only a procedural understanding themselves (1999). Through professional development with the math coach, the teachers gained a deeper understanding of subtraction and their expectation for the students changed. Before implementation of the action plan, the teachers were merely hoping students would improve their subtraction skills. As the teachers' understanding of subtraction evolved, teachers began to expect students to justify their strategy choice through making mathematical arguments. Students were asked to compare and contrast subtraction problems solidifying their understandings of the generalizations. Students were given the opportunity to create problems that fit the student created generalization. Furthermore, Russell et al. asserted that one way teachers learn mathematics is by engaging directly in the mathematical content they are teaching their students (1995). As teachers worked through problems using the concrete and representation strategies, the way they envisioned subtraction changed. At the conclusion of the action plan, one teacher stated, "After teaching the children how to represent and draw their subtraction problems, I find that when I am doing subtraction in 'the real world' I see the base-10 blocks in my mind and find that it is easier to complete subtraction in this manner."

As the teachers' focus shifted from the students' answers to the students' thought process, the teachers' mathematical reasoning improved. In the beginning, the teachers asked the math coach, "[w]hat is the best algorithm to teach?" By the end teachers began to ask, if solutions using different strategies to the same subtraction problem were mathematically equivalent. In order to analyze student thinking, teachers must learn how to follow a mathematical argument and assess its validity (Russell et al. 1995). The change in the questions the teachers asked the math coach signified that the teachers were teaching themselves to read and evaluate students' mathematical arguments. The teachers honed their mathematical understanding through independent and collaborative examination of students' written work samples.

STAGE 5: CHANGE FUTURE PRACTICE

The ultimate goal of the action plan was to increase the number of students who mastered subtracting two, two-digit numbers abstractly. Therefore, the assessment given at the end of the unit required students to solve subtraction problems using the abstract strategy. Narrowing the focus of the assessment allowed teachers to identify the students who still needed targeted small group instruction to subtract abstractly. This data was useful in planning subsequent lessons. However, the teachers and the math coach agreed that the assessment did not give an accurate picture of the students' conceptual understanding of subtraction, their ability to subtract correctly by other strategies, or a student's ability to transfer what they learned to more difficult problems. Since this study, the math coach has been examining ways to allow the use of manipulatives on assessments, the parameters for when this is appropriate, and the effects it will have on evaluating student achievement.

Listening to the students explain their thought process, compare and contrast problems and formulate generalizations for future problem solving proved to be a powerful experience for the group. The group realized they cannot assume that a student can explain a process verbally just because they can perform the steps in the process. The group felt having clear expectations for the students, modeling communication skills and providing opportunities for students to practice communicating in nonthreatening partnerships led to a significant growth in the students' communication of mathematical reasoning. This observation led this teacher to adapt subsequent lessons of different mathematical topics to provide opportunity for discussions that will lead to form their own generalizations.

Finally, the group noticed that the student drew subtraction in different ways. Some pictures represented the minuend and the subtrahend, while others only represented the minuend. All the pictures had some symbol to represent the part of the picture that was being removed. Teachers explored the merits of both styles of picture. Some group members suggested that allowing students to draw pictures without guidance from the teachers gives a better understanding of the students' thought process. Other group members argued that there is a point at which too much detail in the picture begins to interfere with the conceptual understanding that the pictures were suppose to develop. Teachers agreed a second research cycle should be conducted to determine if one type of picture produced a better understanding of subtraction or a more efficient learning process. The group agreed to revisit this issue the following year when they repeated the activities with a new group of students.

CONCLUSION

Replicating steps in an algorithm is not a useful skill if students do not know when it is appropriate to use the algorithm. Students need to develop the discipline of analyzing the characteristics of a problem and making generalizations about problems that can be solved with the same strategy if they are to successfully apply their strategy in future problems. To accomplish this, teachers must make connecting mathematical ideas a priority when planning lessons designed to build conceptual understanding. Repetition of an algorithm alone will not help students to connect a subtraction strategy with the characteristics of a subtraction problem. When teachers take the time to explicitly connect the concrete,

representation and the abstract strategies of subtraction, students benefit from a more complete knowledge structure of subtraction. As teachers develop lessons that focus on making mathematical connections, they can expect that their own mathematical reasoning abilities will improve as well.

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BIOGRAPHICAL NOTE:

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